Fair Dynamic Rationing

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Equitable and efficient rationing: We study the allocative challenges that governmental and nonprofit organizations face when tasked with equitable and efficient rationing of a social good among agents whose needs (demands) realize sequentially and are possibly correlated. As one example, early in the COVID-19 pandemic, the Federal Emergency Management Agency (FEMA) faced overwhelming, temporally scattered, a priori uncertain, and correlated demands for medical supplies from different states. Achieving efficiency alone is easy: a first-come, first-serve policy allocates all of the supply to meet early-arriving demands. However, such a policy can be unfair to patients in states where needs materialize later.

In this paper, we develop a framework for fair dynamic rationing where agents' one-time needs (demands) for a divisible good realize sequentially and can be *arbitrarily* correlated. In particular, upon arrival of each agent's demand, the planner makes an irrevocable decision about their *fill rate* (FR). Toward jointly achieving efficiency and equity, the planner aims to *maximize the minimum* FR, either ex post or ex ante.

To assess the performance of sequential allocation policies, we introduce measures of ex-post and ex-ante fairness guarantees. In defining our notions of such guarantees, we use the minimum FR achievable under deterministic demand as a benchmark to separate the impact of demand stochasticity from the impact of *supply scarcity* (i.e., the expected demand-to-supply ratio). The ex-post (resp. ex-ante) fairness guarantee of a policy serves as a lower bound on the expected minimum (resp. minimum expected) FR that the policy achieves relative to our benchmark under all possible joint demand distributions.

Establishing and achieving upper bounds: In order to gain insight into the difficulty of achieving equity and efficiency in sequential allocation, we first develop upper bounds on the ex-post and ex-ante fairness guarantees achievable by an online policy. These bounds are parameterized by the supply scarcity and the number of agents. Remarkably, we show that a simple, adaptive, and transparent policy called *projected proportional allocation* (PPA) simultaneously achieves our upper bounds on the ex-post and ex-ante fairness guarantees for any set of parameters.

To motivate our policy, consider a hypothetical situation where the demand sequence is known a priori. In that case, the optimal policy is to proportionally allocate the remaining supply based on the current demand and the total future demand. When demand is stochastic, our PPA policy simply replaces all the future random demands by their projected values, namely, their conditional expectations. We obtain the performance guarantees of our policy by inductively designing lower-bound functions on its corresponding value-to-go. Beyond enjoying the best possible guarantees, our PPA policy is practically appealing due to its computational efficiency, interpretability, and transparency. Furthermore, it does not require full distributional knowledge, as it only relies on the first conditional moments of the joint distribution for demand.

Establishing sub-optimality of target-fill-rate policies: In addition to showing that our PPA policy achieves the best possible guarantees, we illustrate the power of adaptivity by studying the class of target-fill-rate (TFR) policies. A TFR policy commits upfront to a fill rate τ, and upon arrival of each agent, it allocates a fraction τ of that agent's demand until it exhausts the supply. We provide a tight bound on the ex-post fairness guarantee of the optimal TFR policy, which can be considerably lower than the corresponding guarantee of our adaptive PPA policy. To characterize the ex-post fairness guarantee of the optimal TFR policy, we establish a rather surprising connection to the literature on monopoly pricing and Bayesian mechanism design which can be of independent interest.

Illustrative case study: To demonstrate the effectiveness of our policy, we conduct a numerical case study motivated by the allocative challenges faced by FEMA. Our simulation results illustrate the superior performance of our PPA policy compared to both its ex-post fairness guarantee and the optimal TFR policy.

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